MA4271 Tutorial Week 4

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Problem 1 *Find the curvature of* $\alpha(t) = (t\log t, t^2, \log t)$ *at the point* $(0,1,0)$*, i.e.* $t = 1$*. Here* log *is the logarithm function with base* e

The curve is defined and smooth when $t \in (0, +\infty)$
We have

$$\begin{aligned}\alpha' &= \left(\log t + 1, 2t, \frac{1}{t}\right)\\ \alpha'' &= \left(\frac{1}{t}, 2, \frac{-1}{t^2}\right)\\ \alpha' \cdot \alpha'' &= \frac{\log t + 1}{t} + 4t + \frac{-1}{t^3}\end{aligned}$$

The curvature is of the form

$$\kappa = \frac{\sqrt{||\alpha'||^2||\alpha''||^2 - |\alpha' \cdot \alpha''|^2}}{||\alpha'||^3}$$

At $t = 1$, we have

$$\begin{aligned}\alpha'(1) &= (1,2,1)\\ \alpha''(1) &= (1,2,-1)\\ \alpha'(1) \cdot \alpha''(1) &= 4\end{aligned}$$

α is smooth and regular at $t = 1$, then

$$\kappa(1) = \frac{\sqrt{(1^2+2^2+1^2)(1^2+2^2+(-1)^2) - |4|^2}}{(1^2+2^2+1^2)^{3/2}} = \frac{\sqrt{30}}{18}$$